An Iterative Hungarian Method to Joint Relay Selection and Resource Allocation for D2D Communications

Taejoon Kim, *Member, IEEE*, and Miaomiao Dong

***Abstract*—The joint relay selection and related subchannel and power allocation problem is investigated for relay-aided device-to-device (D2D) communications underlying cellular networks. We show the optimal power allocation problem can be solved in a closed-form. Considering that the associated relay selection and subchannel assignment problem is *NP-complete*, we devise an iterative technique, the iterative Hungarian method (IHM). Interestingly, numerical results show that the proposed technique can offer near-optimal performance with polynomial complexity.**

***Index Terms*—Device-to-device (D2D) relay, resource allocation, relay selection, iterative Hungarian method (IHM).**

I. Introduction

THE emerging device-to-device (D2D) techniques allow a pair of D2D user equipments (UEs) to directly communicate [1]. In cellular network architecture, two D2D UEs suffering from inferior direct link can resort the idle UE, who has no signal to transmit, to act as D2D relay station (RS). The D2D RS routes D2D transmitter's signal to D2D receiver. The subchannels allocated to cellular UEs (CUEs) can also be reused by the D2D RSs. Compared to the fixed RSs [2], the availability of the D2D RSs provides both the spectrum reuse gain and RS selection gain. To exploit these benefits, recent works devise the protocol architecture [3], single D2D RS selection [4], and joint single D2D RS selection and power allocation [5].

Unlike the fixed RS [2], the D2D RS is peak-power limited, thus, being only capable of covering a small area and serving for at most one D2D pair [3]–[5]. This is completely different, in terms of resource allocation, from the powerful fixed RSs [2]. Furthermore, the limitation of co-channel interference requires that the subchannel of CUE is only shared by one D2D pair [1], [3]–[5]. These constraints eventually formulate the problem into a *3-dimensional matching* problem, in which each D2D pair uniquely matches a subchannel allocated to CUE and a D2D RS to maximize the throughput. The *3-dimensional matching problem* is *NP-complete* [6], and there have been no promising techniques directly dealing with the problem, except for those greedy techniques [7], [8] and the exhaustive search [9].

In this letter, we focus on the multiple D2D RS aided D2D cellular uplink scenario with multiple D2D pairs. The novelty of the work mainly lies in introducing new optimization framework to solve the network throughput maximization problem with the joint D2D RS selection and associated subchannel and power allocation. The inherent optimization problem is not directly solvable. To circumvent the computational hurdle, we decompose the original problem into two subproblems: i) the power allocation problem; ii) the associated RS selection and subchannel assignment problem, the *3-dimensional matching problem*. We first show by relaxing the objective function of the former that the power allocation problem can be solved analytically. We propose an iterative technique, referred here to as the iterative Hungarian method (IHM), to solve the latter problem suboptimally with significantly low complexity, while achieving near-optimal throughput performance.

II. System Model

Consider D2D RS aided D2D uplink OFDMA cellular network constituting one base station (BS), K CUEs, R D2D RSs (i.e., idle UEs), and M D2D pairs. Each CUE k $(1 \le k \le K)$ pre-occupies one orthogonal subchannel n $(1 \le n \le K)$ with each having a bandwidth F and transmits signal to BS with maximum allowed power $p_{UE}^{\max}$. We use (k, n) to represent the mapping between CUE k and the subchannel n. Each D2D pair m $(1 \le m \le M)$, comprising of a D2D transmitter m_t and a D2D receiver m_r, i.e., (m_t, m_r) pair, implements simple two-hop transmissions: i) in the first hop, D2D transmitter m_t sends signal to D2D RS r $(1 \le r \le R)$ by reusing one subchannel allocated to CUE; ii) in the second hop, the D2D RS r forwards the received signal to D2D receiver m_r by using the subchannel utilized at the first hop. The D2D RS adopts Amplify-and-Forward strategy. The BS knows any relevant information for decision-making.

A D2D RS r serves for a D2D pair m, forming a pair (m, r). The CUE whose subchannel is shared by an (m, r) pair opportunistically transmits signal once during the first or the second hop to guarantee the link reliability of the D2D pair, while ensuring its own channel access. Let $h_{a,b}^n$ and $p_{a,b}^n$ be the channel coefficient and the transmit power of the link a–b over the subchannel n, respectively. Denote the symbols transmitted by D2D transmitter and CUE as x_1 and x_2, respectively, where $E[|x_1|^2] = E[|x_2|^2] = 1$.

When CUE transmits signal at the first hop, the transmission of D2D RS is interference-free. The D2D RS, then, transmits signal with the UE maximum power $p_{UE}^{\max}$ to maximize the throughput. The received signal at D2D receiver m_r is

$$Y_1 = p_{UE}^{\max\,\frac{1}{2}} h_{r,m_r}^n \beta_1 \left(p_{m_t,r}^{n\ \frac{1}{2}} h_{m_t,r}^n x_1 + p_{UE}^{\max\,\frac{1}{2}} h_{k,r}^n x_2 + z_1 \right) + z_2,$$

where z_1 and z_2 are the noise terms with complex Gaussian distributed as $\mathcal{CN}(0, \sigma^2)$. The $\beta_1 = (p_{m_t,r}^n |h_{m_t,r}^n|^2 + p_{UE}^{\max} |h_{k,r}^n|^2 + \sigma^2)^{-1/2}$ is the power normalization factor at the D2D RS r. Substituting the β_1 into Y_1, the signal-to-interference-plus-noise-ratio (SINR) of D2D pair m at the D2D receiver m_r is $\Gamma_{m_t,m_r}^n(p_{m_t,r}^n) = \Gamma_{m_t,r}^n(p_{m_t,r}^n)\Gamma_{r,m_r}^n / (\Gamma_{m_t,r}^n(p_{m_t,r}^n) + \Gamma_{r,m_r}^n + 1)$, where $\Gamma_{m_t,r}^n(p_{m_t,r}^n) = p_{m_t,r}^n|$

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$h^n_{m_t,r}|^2/(p^{\max}_{UE}|h^n_{k,r}|^2+\sigma^2)$ and $\Gamma^n_{r,m_r}=p^{\max}_{UE}|h^n_{r,m_r}|^2/\sigma^2$. We will define $\Gamma^n_{k,B}(p^n_{m_t,r})=p^{\max}_{UE}|h^n_{k,B}|^2/(p^n_{m_t,r}|h^n_{m_t,B}|^2+\sigma^2)$ as the SINR of CUE k at the BS, where the subscript B denotes the BS.

To ensure the reliable transmission, the SINRs of the CUEs and D2D pairs must be larger than the threshold Γ_t, i.e., $\min(\Gamma^n_{m_t,r}(p^n_{m_t,r}),\Gamma^n_{m_t,m_r}(p^n_{m_t,r}),\Gamma^n_{k,B}(p^n_{m_t,r}))\geq\Gamma_t$, yielding the power constraint for $p^n_{m_t,r}$, $c1: p^L_1\leq p^n_{m_t,r}\leq p^U_1,\ \forall\ m,r,n$, where $p^L_1=\max((\Gamma_t(1+\Gamma^n_{r,m_r})/(\Gamma^n_{r,m_r}-\Gamma_t)A_1),(\Gamma_t/A_1))$ with $A_1=|h^n_{m_t,r}|^2/(p^{\max}_{UE}|h^n_{k,r}|^2+\sigma^2)$ and $p^U_1=\min(p^{\max}_{UE},(p^{\max}_{UE}|h^n_{k,B}|^2/\Gamma_t|h^n_{m_t,B}|^2)-(\sigma^2/|h^n_{m_t,B}|^2))$.

When CUE transmits signal at the second hop, since the transmission of D2D transmitter is interference-free, the $p^n_{m_t,r}$ is set to $p^{\max}_{UE}$ to maximize the throughput. The received signal at D2D receiver m_r is

$$Y_2=p^n_{r,m_r}{}^{\frac{1}{2}}h^n_{r,m_r}\beta_2\left(p^{\max}_{UE}{}^{\frac{1}{2}}h^n_{m_t,r}x_1+z_1\right)+p^{\max}_{UE}{}^{\frac{1}{2}}h^n_{k,m_r}x_2+z_2,$$

where the $\beta_2=(p^{\max}_{UE}|h^n_{m_t,r}|^2+\sigma^2)^{-1/2}$ is the power normalization factor at the D2D RS r. Substituting the β_2 into Y_2, the SINR of D2D pair m at the D2D receiver m_r is $\gamma^n_{m_t,m_r}(p^n_{r,m_r})=\gamma^n_{m_t,r}\gamma^n_{r,m_r}(p^n_{r,m_r})/(\gamma^n_{m_t,r}+\gamma^n_{r,m_r}(p^n_{r,m_r})+1)$ with $\gamma^n_{m_t,r}=p^{\max}_{UE}|h^n_{m_t,r}|^2/\sigma^2$ and $\gamma^n_{r,m_r}(p^n_{r,m_r})=p^n_{r,m_r}|h^n_{r,m_r}|^2/(p^{\max}_{UE}|h^n_{k,m_r}|^2+\sigma^2)$. The SINR of CUE k is given by $\gamma^n_{k,B}(p^n_{r,m_r})=p^{\max}_{UE}|h^n_{k,B}|^2/(p^n_{r,m_r}|h^n_{r,B}|^2+\sigma^2)$.

The reliable link between CUEs and D2D must meet $\min(\gamma^n_{r,m_r}(p^n_{r,m_r}),\gamma^n_{m_t,m_r}(p^n_{r,m_r}),\gamma^n_{k,B}(p^n_{r,m_r}))\geq\Gamma_t$, leading to the power constraint for p^n_{r,m_r}, $c2: p^L_2\leq p^n_{r,m_r}\leq p^U_2$, $\forall m,r,n$, where $p^L_2=\max((\Gamma_t(1+\Gamma^n_{m_t,r})/(\Gamma^n_{m_t,r}-\Gamma_t)A_2),(\Gamma_t/A_2))$ with $A_2=|h^n_{r,m_r}|^2/(p^{\max}_{UE}|h^n_{k,m_r}|^2+\sigma^2)$ and $p^U_2=\min(p^{\max}_{UE},(p^{\max}_{UE}|h^n_{k,B}|^2/\Gamma_t|h^n_{r,B}|^2)-(\sigma^2/|h^n_{r,B}|^2))$.

By Shannon formula $r(x)=F\log_2(1+x)$, the throughputs of link a–b over the subchannel n can be calculated by $r^n_{a,b}(x)=F\log_2(1+\Gamma^n_{a,b}(x))$ or $r^n_{a,b}(x)=F\log_2(1+\gamma^n_{a,b}(x))$, depending on in which hop the CUE transmits signal.

III. Problem Formulation and Decomposition

Denote by (m,r,n), the set of indices of D2D pair m, D2D RS r, and subchannel n. For every (m,r,n), the sum rate of D2D pair m and CUE k after the two-hop transmission is

$$R^{(k,n)}_{m,r}=\frac{1}{2}\max\left(r^n_{k,B,m_t,m_r}\left(p^n_{m_t,r}\right),r^n_{k,B,m_t,m_r}\left(p^n_{r,m_r}\right)\right),\tag{1}$$

where $r^n_{k,B,m_t,m_r}(p)=r_{k,B}(p)+r_{m_t,m_r}(p)$.

Provided (1), the joint RS selection and associated subchannel and power allocation problem is now formulated as

$$\begin{aligned}&(\text{P1}):\max_{\mathbf{p}}\max_{\mathbf{X}}\sum_{m=1}^{M}\sum_{r=1}^{R}\sum_{n=1}^{K}x^n_{m,r}R^{(k,n)}_{m,r}\\&\text{s.t. } c1: p^L_1\leq p^n_{m_t,r}\leq p^U_1, c2: p^L_2\leq p^n_{r,m_r}\leq p^U_2,\ \forall m,r,n,\\&\quad c3:\sum_{m=1}^{M}\sum_{n=1}^{K}x^n_{m,r}=1,\ \forall r;\ \sum_{m=1}^{M}\sum_{r=1}^{R}x^n_{m,r}=1,\ \forall n,\end{aligned}\tag{2}$$

where $\mathbf{X}_{M\times R\times K}\in\mathbb{R}^{M\times R\times K}$ is the 3-dimensional allocation matrix with the (m,r,n)th entry being $x^n_{m,r}\in\{0,1\}$. The $x^n_{m,r}=1$ if D2D pair m selects D2D RS r to relay signal by reusing subchannel n, and $x^n_{m,r}=0$, otherwise. The mth element of $\mathbf{p}_{M\times 1}\in\mathbb{R}^{M\times 1}$ is either $p^n_{m_t,r}$ or p^n_{r,m_r}, determined by (1). The constraint $c3$ indicates that a D2D RS can only serve for one D2D pair, and a CUE can only share its subchannel with one (m,r) pair.

Solving for the optimal $\mathbf{X}^\star$ and $\mathbf{p}^\star$ in (P1) is a mixed integer programming problem, which is extremely complex to solve. To make it tractable, we first attempt to decompose (P1). Since the objective function in (P1) consists of two maximizations, the order of the maximizations can be switched to $\max_{\mathbf{X}}\sum_{m=1}^{M}\sum_{r=1}^{R}\sum_{n=1}^{K}x^n_{m,r}\max_{\mathbf{p}}R^{(k,n)}_{m,r}$ s.t. $c1,c2,c3$. Since the constraints $c1$, $c2$, and $c3$ are *separable*, we can decompose (P1) into two subproblems, the *power allocation subproblem*

$$(\text{P2}):\quad R^{(k,n)\star}_{m,r}=\max_{\mathbf{p}}\quad R^{(k,n)}_{m,r}\quad\text{s.t.}\quad c1,c2,\tag{3}$$

and the *3-dimensional matching problem*

$$(\text{P3}):\Phi(\mathbf{X}^\star)=\max_{\mathbf{X}}\sum_{m=1}^{M}\sum_{r=1}^{R}\sum_{n=1}^{K}x^n_{m,r}R^{(k,n)\star}_{m,r}\quad\text{s.t.}\quad c3.\tag{4}$$

IV. Throughput Maximization: Algorithm Design

We present, in this section, the algorithms that solve (P2) and (P3) sequentially. The solution to (P2) is first discussed.

From (1), the (P2) is further divided into two subproblems

$$H\left(p^{n^\star}_{m_t,r}\right)=\max_{p^n_{m_t,r}}\left(1+\Gamma^n_{m_t,m_r}\left(p^n_{m_t,r}\right)\right)\times\left(1+\Gamma^n_{k,B}\left(p^n_{m_t,r}\right)\right)\text{s.t. } c1\tag{5}$$

and

$$G\left(p^{n^\star}_{r,m_r}\right)=\max_{p^n_{r,m_r}}\left(1+\gamma^n_{m_t,m_r}\left(p^n_{r,m_r}\right)\right)\times\left(1+\gamma^n_{k,B}\left(p^n_{r,m_r}\right)\right)\text{s.t. } c2,\tag{6}$$

where we use the fact that $\arg\max_x(F/2)(\log_2 f(x)+\log_2 g(x))$ is equivalent to $\arg\max_x f(x)g(x)$. Solving (5) and (6) yields $R^{(k,n)^\star}_{m,r}=(F/2)\log_2(\max(H(p^{n^\star}_{m_t,r}),G(p^{n^\star}_{r,m_r})))$. The following Lemma discusses finding the $p^{n^\star}_{m_t,r}$ in (5) over the interval $c1$.

Lemma 1: The optimal transmit power $p^{n^\star}_{m_t,r}$ that achieves $H(p^{n^\star}_{m_t,r})$ in (5) is a boundary point of the constraint $c1$.

Proof: Evaluating the first- and second-order derivatives of (5) with respect to $p^n_{m_t,r}$ verifies that the objective function $(1+\Gamma^n_{m_t,m_r}(p^n_{m_t,r}))(1+\Gamma^n_{k,B}(p^n_{m_t,r}))$ is either monotone or convex over $p^n_{m_t,r}$, depending on the channel coefficients. Notice that the maximum of any monotone or convex function is achieved at the boundary points of its domain. Hence, the optimal $p^{n^\star}_{m_t,r}$ in (5) is found by evaluating the objective function at two points $p^{n^\star}_{m_t,r}\in\{p^L_1,p^U_1\}$ of $c1$ in (2) and taking the one resulting in the larger objective function value. This concludes the proof. ■

Since the structure of the objective functions in (6) is same as (5), Lemma 1 can be reused to calculate the $p^{n^\star}_{r,m_r}$ in (6).

Provided the $R^{(k,n)^\star}_{m,r}$ from (P2), we now solve for the (P3). As aforementioned in Section III, (P3) is the *3-dimensional matching problem*, which is *NP-complete*. Hence, the primary challenge we face here is how to devise an algorithm that solves the (P3) within an acceptable computational overhead, while achieving close-to-optimal performance. We resolve this challenge by proposing an iterative technique which projects the *3-dimensional matching problem* in (P3) to an iterative 2-dimensional matching problems.

Suppose an arbitrary initial allocation matrix $\mathbf{X}_{M\times R\times K} = \mathbf{X}_0^\star$ which is feasible to (P3), i.e., satisfying the constraint $c3$. Let a 2-dimensional index set comprising of indices of D2D pair and D2D RS be $\mathcal{X}_0 = \{(m,r)|[\mathbf{X}_0^\star]_{m,r,n} = 1,\ \forall m,r,n\}$. For notational simplicity, we introduce an index d to indicate the pair $(m,r) \in \mathcal{X}_0$ such that $d = mu(R-M) + ru(M-R)$, where the $u(t)$ is the unit step function defined according to $u(t) = 1$ if $t \geq 0$, and $u(t) = 0$, otherwise. For instance, if $M \leq R$, the $d = m$ represents the pair (m,r), and the $d = r$ indicates the (m,r), otherwise. We adopt the 2-dimensional matching matrix $\mathbf{T}_{0,\min(M,R)\times K} = [t_d^n]$ with $t_d^n = 1$ if the subchannel n is assigned to the pair $(m,r) \in \mathcal{X}_0$, otherwise $t_d^n = 0$. We will write $\mathbf{X}_0^\star = (\mathcal{X}_0, \mathbf{T}_0)$. The problem of designing the optimal subchannels allocation $\mathbf{T}_0^\star$ to $\mathcal{X}_0$ is formulated to

$$\begin{aligned}(\text{P3.1}): \Phi\left(\mathcal{X}_0, \mathbf{T}_0^\star\right) &= \max_{\mathbf{T}_0} \sum_{d=1}^{\min(M,R)} \sum_{n=1}^{K} t_d^n R_{d,n}^\star \\ \text{s.t.} &\sum_{d=1}^{\min(M,R)} t_d^n = 1, \forall n; R_{d,n}^\star = R_{m,r}^{(k,n)\star}, \forall (m,r) \in \mathcal{X}_0. \quad (7)\end{aligned}$$

The constraint indicates that one subchannel can uniquely be allocated to the one pair $(m,r) \in \mathcal{X}_0$. The problem at hand is now the 2-dimensional matching problem. The modified Hungarian method [10] is already available to solve the (P3.1) optimally and yields $\mathbf{X}_1^\star = (\mathcal{X}_0, \mathbf{T}_0^\star)$ which is feasible to (P3).

Next, the index set of D2D RS and subchannel is extracted, $\mathcal{X}_1 = \{(r,n)|[\mathbf{X}_1^\star]_{m,r,n} = 1, \forall m,r,n\}$. Similar to (7), we introduce an index i to indicate the (r,n) such that $i = ru(K-R) + nu(R-K)$. The 2-dimensional matching matrix $\mathbf{S}_{1,\min(R,K)\times M} = [s_i^m]$ is defined, where $s_i^m = 1$ if the D2D pair m is allocated to the pair (r,n), and $s_i^m = 0$, otherwise. Thus, $\mathbf{X}_1^\star = (\mathcal{X}_1, \mathbf{S}_1)$. The optimal D2D pair allocation $\mathbf{S}_1^\star$ to the set $\mathcal{X}_1$ is found by solving

$$\begin{aligned}(\text{P3.2}): \Phi\left(\mathcal{X}_1, \mathbf{S}_1^\star\right) &= \max_{\mathbf{S}_1} \sum_{i=1}^{\min(R,K)} \sum_{m=1}^{M} s_i^m R_{i,m}^\star \\ \text{s.t.} &\sum_{i=1}^{\min(R,K)} s_i^m = 1, \forall m; R_{i,m}^\star = R_{m,r}^{(k,n)\star}, \forall (r,n) \in \mathcal{X}_1. \quad (8)\end{aligned}$$

This problem has the same structure as (7). Solving (P3.2) returns the (P3) feasible solution $\mathbf{X}_2^\star = (\mathcal{X}_1, \mathbf{S}_1^\star)$.

Subsequently, we take the index set of subchannel and D2D pair, $\mathcal{X}_2 = \{(n,m)|[\mathbf{X}_2^\star]_{m,r,n} = 1, \forall m,r,n\}$. Define the index $\ell = mu(K-M) + nu(M-K)$ and the 2-dimensional matching matrix $\mathbf{Z}_{2,\min(K,M)\times R} = [z_\ell^r]$, which represents the matching between D2D RSs and the pairs $(n,m) \in \mathcal{X}_2$. The optimal D2D RS selection $\mathbf{Z}_2^\star$ for the set $\mathcal{X}_2$ is attained by solving

$$\begin{aligned}(\text{P3.3}): \Phi\left(\mathcal{X}_2, \mathbf{Z}_2^\star\right) &= \max_{\mathbf{Z}_2} \sum_{\ell=1}^{\min(K,M)} \sum_{r=1}^{R} z_\ell^r R_{\ell,r}^\star \\ \text{s.t.} &\sum_{\ell=1}^{\min(K,M)} z_\ell^r = 1,\ \forall r;\ R_{\ell,r}^\star = R_{m,r}^{(k,n)\star},\ \forall (n,m) \in \mathcal{X}_2. \quad (9)\end{aligned}$$

The modified Hungarian method yields $\mathbf{X}_3^\star = (\mathcal{X}_2, \mathbf{Z}_2^\star)$, which is also feasible to (P3).

The iteration continues by solving (P3.1) with the updated $\mathcal{X}_3 = \{(m,r)|[\mathbf{X}_3^\star]_{m,r,n} = 1, \forall m,r,n\}$. We implement the subsequent iteration $\mathbf{X}_0^\star \to \mathbf{X}_1^\star \to \mathbf{X}_2^\star \to \mathbf{X}_3^\star \ldots$. Herein, the proposed procedure is referred to as the iterative Hungarian method (IHM).

Theorem 1: The sequence $\{\Phi(\mathbf{X}_j^\star)\}_{j\geq 0}$ of IHM is non-decreasing and converges to a stationary solution of (P3), when $\mathbf{X}_{j+1}^\star = \mathbf{X}_{j+2}^\star = \mathbf{X}_{j+3}^\star, j \geq 0$.

Proof: Suppose the jth iteration with $\mathrm{mod}(j,3) = 0$, where $\mathrm{mod}(j,3)$ is the modulo 3 of j. The case when $\mathrm{mod}(j,3) \neq 0$ can be handled similarly to the case of $\mathrm{mod}(j,3) = 0$. It can be seen, for instance, from (P3.1), that $\Phi(\mathbf{X}_j^\star) = \Phi(\mathcal{X}_j, \mathbf{T}_j) \leq \max_{\mathbf{T}_j} \Phi(\mathcal{X}_j, \mathbf{T}_j) = \Phi(\mathcal{X}_j, \mathbf{T}_j^\star) = \Phi(\mathbf{X}_{j+1}^\star)$. Then, it follows that $\Phi(\mathbf{X}_{j+1}^\star) = \Phi(\mathcal{X}_{j+1}, \mathbf{S}_{j+1})$. In the next iteration, we have that $\Phi(\mathbf{X}_{j+1}^\star) = \Phi(\mathcal{X}_{j+1}, \mathbf{S}_{j+1}) \leq \max_{\mathbf{S}_{j+1}} \Phi(\mathcal{X}_{j+1}, \mathbf{S}_{j+1}) = \Phi(\mathcal{X}_{j+1}, \mathbf{S}_{j+1}^\star) = \Phi(\mathbf{X}_{j+2}^\star)$. Following an exactly same reasoning, $\Phi(\mathbf{X}_{j+2}^\star) = \Phi(\mathcal{X}_{j+2}, \mathbf{Z}_{j+2}) \leq \max_{\mathbf{Z}_{j+2}} \Phi(\mathcal{X}_{j+2}, \mathbf{Z}_{j+2}) = \Phi(\mathcal{X}_{j+2}, \mathbf{Z}_{j+2}^\star) = \Phi(\mathbf{X}_{j+3}^\star)$. Finally, combining the three steps yields $\Phi(\mathbf{X}_j^\star) \leq \Phi(\mathbf{X}_{j+1}^\star) \leq \Phi(\mathbf{X}_{j+2}^\star) \leq \Phi(\mathbf{X}_{j+3}^\star)$. Thus, the recursive application verifies that the sequence $\{\Phi(\mathbf{X}_j^\star)\}_{j\geq 0}$ is non-decreasing. Suppose now that $\mathbf{X}_{j+1}^\star = (\mathcal{X}_j, \mathbf{T}_j^\star)$ and the convergence condition $\mathbf{X}_{j+1}^\star = \mathbf{X}_{j+2}^\star = \mathbf{X}_{j+3}^\star$, $j \geq 0$ is satisfied. The index set $\mathcal{X}_{j+3}$ is extracted from $\mathbf{X}_{j+3}^\star$, i.e., $\mathbf{X}_{j+3}^\star = (\mathcal{X}_{j+3}, \mathbf{T}_{j+3})$ and then, the algorithm yields $\max_{\mathbf{T}_{j+3}} \Phi(\mathcal{X}_{j+3}, \mathbf{T}_{j+3}) = \Phi(\mathcal{X}_{j+3}, \mathbf{T}_{j+3}^\star)$. However, since $\max_{\mathbf{T}_j} \Phi(\mathcal{X}_j, \mathbf{T}_j) = \Phi(\mathcal{X}_j, \mathbf{T}_j^\star)$ and $\mathcal{X}_j = \mathcal{X}_{j+3}$ (because $\mathbf{X}_{j+1}^\star = \mathbf{X}_{j+3}^\star$), it immediately follows that $\mathbf{T}_{j+3}^\star = \mathbf{T}_j^\star$, i.e., $(\mathcal{X}_j, \mathbf{T}_j^\star) = (\mathcal{X}_{j+3}, \mathbf{T}_{j+3}^\star)$. This shows that if the convergence condition holds, the IHM returns the sequence $\{\Phi(\mathbf{X}_l^\star)\}_{l\geq j+1}$ keeping constant in subsequent iterations. ■

The sequence of iteration generated by the IHM converges to at least a local optima of the throughput maximization problem, and does so with low complexity as shown in the next section. We remark that both the IHM and its convergence result (Theorem 1) can be extended to any 3-dimensional matching problems.

V. Simulation Results

In this section, we present the simulation results to demonstrate the performance of the proposed algorithm. We consider a cell with radius 200 m. The CUEs, D2D RSs, and D2D pairs are randomly drawn in the cell. To emulate practical channel propagation, the LTE typical urban channel model is employed [11]. The white noise power density is -158 dBm/Hz and $F = 180$ KHz. The maximum allowed UE and D2D RS transmit power is $p_{UE}^{\max} = 23$ dBm. It is noteworthy that the transmit power of D2D RS is substantially lower than that of the fixed RS (e.g., 30 dBm [2]). We assume $R = 8$ and $M = 4$. For the comparison, we take the following three benchmark schemes: greedy [7], improved greedy [8], and optimal scheme [9].

A. Greedy and Improved Greedy Algorithms

In the absence of the power allocation scheme for the greedy [7] and the improved greedy [8], we set $p_{m_r,r}^n = p_{r,m_r}^n = p_{UE}^{\max}$. In the greedy algorithm [7], each D2D pair sequentially and exclusively selects the D2D RS and subchannel pair such that $(r^\star, n^\star) = \arg\max_{(r,n)} R_{m,r}^{(k,n)}$. The

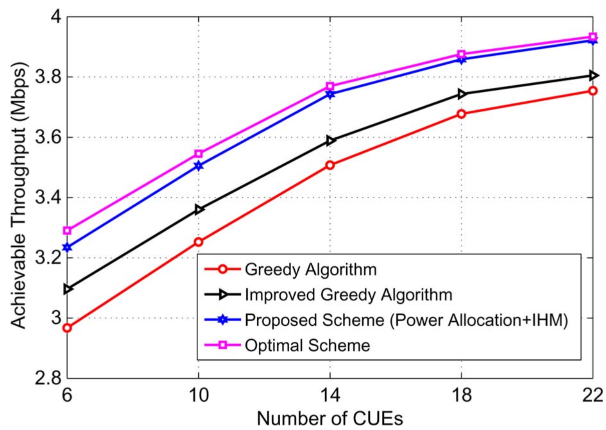

Fig. 1. Throughput comparison of different schemes vs. number of CUEs.

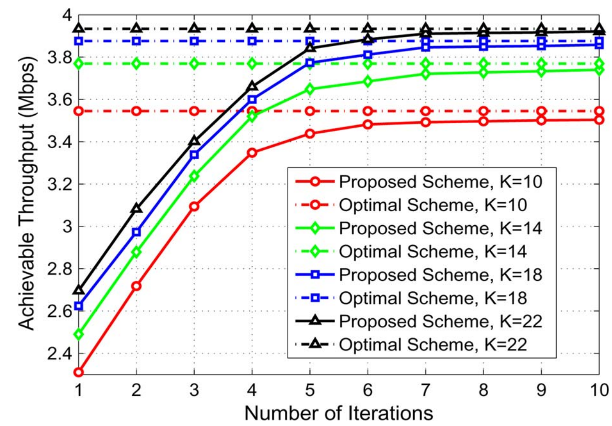

Fig. 2. Throughput comparison of the IHM vs. different number of iterations.

improved greedy algorithm [8] first calculates the achievable maximum rate of each D2D pair and sorts them in the descending order. Then, the D2D pair sequentially and exclusively selects the D2D RS and subchannel pair such that $(r^\star, n^\star) = \arg\max_{(r,n)} R^{(k,n)}_{[m],r}$, where the $[m]$ denotes the computed order. The complexities of greedy algorithm and improved greedy algorithm are $O(MRK)$ [7] and $O(MRK + M \log M)$ [8], respectively, where the complexity is measured by the number of objective function evaluations.

B. Optimal Scheme

The optimal scheme is the one that directly solves (P3) given $R^{(n,k)\star}_{m,r}$ in (3). The complexity of the exhaustive search [9] (complexity $O(R^M K!)$) is not practically manageable. We propose instead the *improved exhaustive search*. The proposed scheme first finds all of the possible 2-dimensional matchings (i.e., permutations) between M D2D pairs and R D2D RSs. There are total $P^{\max(M,R)}_{\min(M,R)} = \max(M,R)!/((\max(M,R) - \min(M,R))!)$ permutations (or matchings). Given each matching, we optimally solve the (P3.1) by applying the modified Hungarian method. Then, the solution with the maximum throughput is the optimal solution of (P3). The complexity of the modified Hungarian method is $O(L^3)$, where $L = \max(M,R,K)$ [10]. Overall, the complexity of the *improved exhaustive search* is $O(L^3 P^{\max(M,R)}_{\min(M,R)}) (\ll O(R^M K!))$.

Fig. 1 compares the achievable throughput of the proposed scheme (power allocation+IHM) with three other benchmarks across different numbers of CUEs. It should be mentioned that the IHM depends on the initial matching $\mathbf{X}_0^\star$. For the proposed scheme, we run the IHM 15 times with random $\mathbf{X}_0^\star$ initializations, and keep the best result. It can be seen from Fig. 1 that the IHM with random initialization shows the close-to-optimal performance. Moreover, the throughput of the proposed scheme increases and indeed converges to the optimal performance as the number of CUE increases, while the gaps between the other approaches and the optimal scheme keep almost constant as the number of CUEs increases. Fig. 2 illustrates the convergence behavior of the proposed scheme under different numbers of CUEs. The throughput grows as the number of iterations increases and converges, for instance, after 5 times of iterations. The IHM with j iterations has the complexity $O(jL^3)$, $L = \max(M,R,K)$. Figs. 1 and 2 at least illustrate that the proposed IHM shows the near-optimal performance with substantially reduced computational complexity (polynomial time) compared to the exhaustive search (i.e., $O(jL^3) \ll O(L^3 P^{\max(M,R)}_{\min(M,R)}) \ll O(R^M K!)$).

VI. Conclusion

We have addressed the joint RS selection and related subchannel and power allocation for the D2D RS aided D2D communication cellular uplink network with multiple D2D pairs, D2D RSs, and CUEs. The original throughput maximization problem is *NP-complete*. The proposed approach interestingly shows near-optimal performance with practical polynomial complexity ($O(L^3)$) and outperforms the other greedy schemes. However, it has the dependency on the initial condition. Investigating a technique to find a suitable initial condition is subject to further research.

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